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Multiple muons deep underground†

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Abstract. A sample of 176 723 single cosmic ray muon events, 2802 two-muon events, and 225 three-muon events have been collected and analysed to obtain muon rates adjusted for an equivalent detector having a constant normal area of 20 m^2. The measurements span the range of zenith angles between 40 and 70° and slant depths of standard rock between 2×10^5 g cm^{-2} and $6{\cdot}5 \times 10^5$ g cm^{-2}, corresponding to minimum muon energies at ground level of between 750 and 8000 GeV. A comparison of these measurements with those predicted by a conservative model based upon extrapolation from results at lower energies is given in the following paper by Adcock *et al.*

1. Introduction

Experimental data on cosmic ray muon showers detected underground are examined, the term 'shower' here referring to a bundle of very nearly parallel muons which are typically spread over an area of 100 to 300 m^2. Because of their parallelism and simultaneous detection, all muons in the shower are presumed to be the result of atmospheric interactions caused by a single cosmic ray primary and its progeny. The measurements presented in this paper are compared in the following paper by Adcock *et al.* (1971 to be referred to as II) with the detailed predictions of a conservative theory based on an extrapolation of results on high energy interactions valid at lower energies. Comparison with this theory should give some indication of the behaviour of interaction processes at the high primary energies (10^5–10^6 GeV) responsible for the detected events.

In so far as the results accumulated are statistically the most precise to date, the basic data and their combination are given in some detail.

2. Shower measurements at Utah

2.1. *The detector*

The University of Utah operates a large underground detector (figure 1) located in the Wasatch Mountains near Park City, Utah. Its large size makes it particularly useful as a detector for showers of muons (Porter *et al.* 1969). The apparatus consists of 15 vertical planes of spark counters arranged in 8 groups which are triggered by a coincidence betwen pairs of the four water Čerenkov counters.

In order for a muon trajectory to be counted as 'in aperture', it must pass through at least two Čerenkov tanks and three groups of spark counters (see figure 1) ensuring high triggering efficiency and great spatial resolution. A muon is counted as having passed through a Čerenkov tank only if it passes no closer than one foot from the edge of the forward wall (as seen by the incoming muon) of the tank and goes through the backward wall as well. Every particle present in an event is used to calculate the triggering efficiency of the event; however, only those satisfying the criteria above are counted as being in the allowable aperture. The categories are defined in the following way. An event which has one muon in the aperture above is included in the single

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muon rate, two muons in the rate of twos, and three muons in the rate of threes. This convention is different from that used previously by the Utah group (Bergeson *et al.* 1967, 1968) who classified an event having three muons in their aperture as comprising *three single muons*.

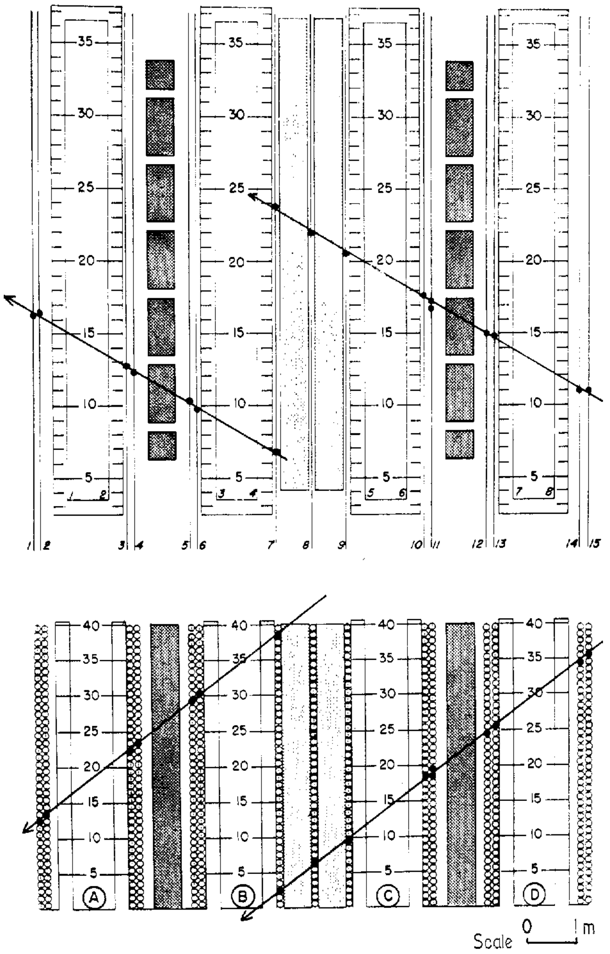

Figure 1. Front (XZ plane) and top (XY plane) views of the University of Utah detector. In the front view, cylindrical spark counters are seen end-on as circles stacked in columns 40 high on either side of water-filled Čerenkov tanks labelled A, B, C and D. The dark cross-hatched areas between A and B and between C and D are the solid iron magnets. The light dotted areas between B and C are concrete blocks. In the top view, columns of sonic cylindrical spark counters appear as lines labelled 1 to 15, and the light-collecting walls of the Čerenkov tanks are labelled 1 to 8. The eight groups of spark chambers are the columns that follow: (1 and 2), (3 and 4), (5 and 6), (7, 9), (10 and 11), (12 and 13), (14 and 15).

The data from the detector consist of arrays of numbers of events as a function of zenith and azimuth. A careful geological survey giving the slant depth of standard rock as a function of zenith and azimuth was used to produce arrays giving the distribution of events in twelve 2·5° zenith bins beginning at 40° and twelve 5×10^4 g cm^{-2} depth bins beginning at 2×10^5 g cm^{-2} (see tables 1, 2 and 3).

Table 1. Distribution of numbers of single muons as a function of zenith angle and depth

Multiplicity = 1 Zenith (deg)	Slant depth (10^5 g cm^{-2})									
	(2·25)	(2·75)	(3·25)	(3·75)	(4·25)	(4·75)	(5·25)	(5·75)	(6·25)	Total
(41·25)	8423	1323								9746
(43·75)	8919	4965								13884
(46·25)	6995	9743								16738
(48·75)	5022	11051	3014							19087
(51·25)	713	15470	4897							21080
(53·75)	14	15104	6355							21473
(56·25)		10377	8135	1151						19663
(58·75)		4229	7552	4894	443					17118
(61·25)			9194	4347	669	132				14342
(63·75)			4426	4528	1477	284	60			10775
(66·25)			1360	3605	1410	1104	105	48		7632
(68·75)				1898	1810	958	350	136	33	5185
Total	30086	72262	44933	20423	5809	2478	515	184	33	

Table 2. Distributions of numbers of twos as a function of zenith and depth

Multiplicity = 2

Zenith (deg)	Slant depth (10^5 g cm^{-2})									
	(2·25)	(2·75)	(3·25)	(3·75)	(4·25)	(4·75)	(5·25)	(5·75)	(6·25)	Total
(41·25)	59	8								67
(43·75)	110	64								174
(46·25)	88	168								256
(48·75)	69	139	48							256
(51·25)	8	271	93							372
(53·75)		289	125							414
(56·25)		176	112	24						312
(58·75)		71	130	95	5					301
(61·25)			140	93	14	2				249
(63·75)			80	81	25	3	2			191
(66·25)			29	60	20	14	3	1		127
(68·75)				22	33	14	10	3	1	83
Total	334	1186	757	375	97	33	15	4	1	

Table 3. **Distribution of numbers of threes as a function of zenith and depth**

Multiplicity = 3 Zenith (deg)	Slant depth (10^5 g cm^{-2})									
	(2·25)	(2·75)	(3·25)	(3·75)	(4·25)	(4·75)	(5·25)	(5·75)	(6·25)	Total
(41·25)	3	1								4
(43·75)	4	3								7
(46·25)	6	10								16
(48·75)	7	17	7							31
(51·25)		20	13							33
(53·75)		21	15							36
(56·25)		20	14	2						36
(58·75)		11	15	10						36
(61·25)			18	10						28
(63·75)			5	9	3					17
(66·25)				5	2					7
(68·75)				1		3				4
Total	20	103	87	37	5	3				

2.2. *Single muon intensities and combination of data*

In order to calculate intensities we need to determine the effective aperture in each zenith-depth bin. Let $N_{ik'}'$ be the number of muons in the ith zenith bin and the kth depth bin, corrected for detector triggering efficiencies. Then

$$I_{ik'} = \frac{N_{ik'}'}{(A\Omega)_{ik'}t}$$

where I is the muon intensity during time t in the aperture $A\Omega$ (i.e. detector sensitive area A times solid angle Ω).

Now $N_{ik'}' = \Sigma_j N_{ij}'$, where N_{ij}' is the actual number of muons, corrected for detector triggering efficiencies, in the ith zenith bin and the jth azimuth bin which fall in the same range of depths covered by the kth depth bin.

This is also equal to

$$\sum_j I_{ij}(A\Omega)_{ij}t$$

and can be written as

$$\sum_j I_{ik'}\frac{I_{ij}}{I_{ik'}}(A\Omega)_{ij}t = I_{ik'}\sum_j \frac{I_{ij}}{I_{ik'}}(A\Omega)_{ij}t \simeq I_{ik'}\sum_j \frac{I_{\mathrm{v}ij}}{I_{\mathrm{v}k'}}(A\Omega)_{ij}t.$$

Here I_v is the intensity according to the vertical depth intensity curve (see figure 2).

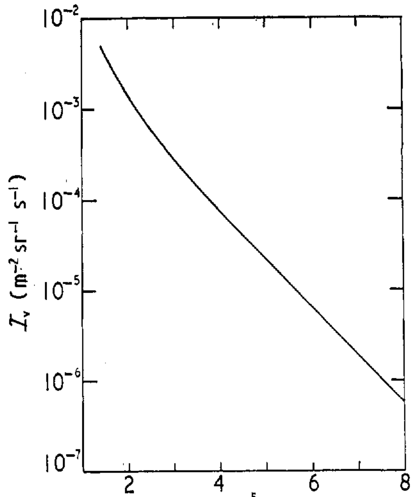

Figure 2. Utah vertical depth intensity curve for single muons.

The indices k', i, and j refer to the depth at the centre of the kth depth bin and the particular depth corresponding to the ith zenith bin and the jth azimuth bin which falls in the range of depths spanned by k'. Our approximation makes use of the empirical fact that the uncorrected intensities exhibit roughly the same relative attenuation with depth as shown by I_v.

Hence, the effective apertures corrected for the distribution of depths contributing to each depth bin are given by

$$(A\Omega)_{ik'} = \sum_j \frac{I_{\mathrm{v}ij}}{I_{\mathrm{v}k'}}(A\Omega)_{ij}.$$

2.3. *Correcting the multiple muon intensities for the relative size of the shower and the detector*

In so far as the rate of multiple events is not proportional to area (e.g. for twos and a small area the rate is proportional to A^2) the term 'intensity' must be used with care. We define 'intensity' as the rate of multiples for a given standard area (20 m^2) divided by 20.

The above aperture array is valid only for single-muon events—for multiple events an additional correction has to be made taking into account the fact that the detector will in general see only a portion of a shower. What is important for events having more than one muon is the relative size of the shower and the detector sensitive area. Suppose the shower covers an area S and the detector size is A. The probability of two particles falling into A is proportional to A^2, if A is small, and proportional to A if A is large compared with the size of the shower. In a similar way, the probability of three particles falling into A is proportional to A^3, if A is small, and to A if A is large compared with the size of the shower.

To obtain a qualitative feeling for the dependence of shower size on zenith and muon energy, consider showers that are produced at a fixed interaction height h. Suppose further that all muons produced have a longitudinal energy E_L and a transverse energy E_T. Then by similar triangles, the shower radius r is related to these quantities by the relation

$$\frac{r}{h\sec\theta}=\frac{E_T}{E_L}=\frac{p_T c}{E_L} \quad \text{or} \quad r \propto p_T\left(\frac{\sec\theta}{E_L}\right)$$

where p_T is the transverse momentum of the muons. It is shown in II that the distribution of production heights and other factors modify the above dependence so that it becomes $\langle r\rangle \propto \langle p_T\rangle \sec^{1\cdot3}\theta E_L{}^{-0\cdot8}$. Then

$$S \propto \langle r\rangle^2 \propto \langle p_T\rangle^2 \sec^{2\cdot6}\theta\, E_L{}^{-1\cdot6}$$

$\langle p_T\rangle$ now being the mean transverse momentum and $\langle r\rangle$ the mean radius.

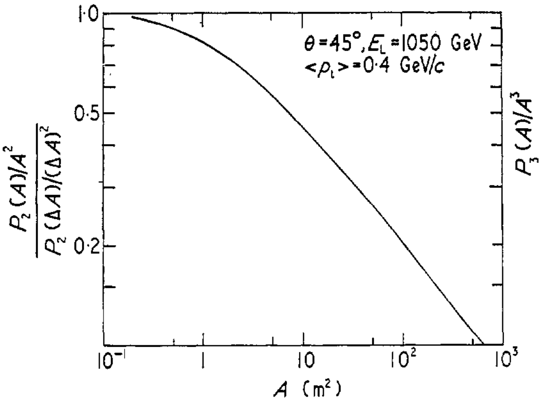

Figure 3. Area-dependence of the probability of detecting two muons (after II). $P_2(A)$ is the probability of detecting two muons in an area A.

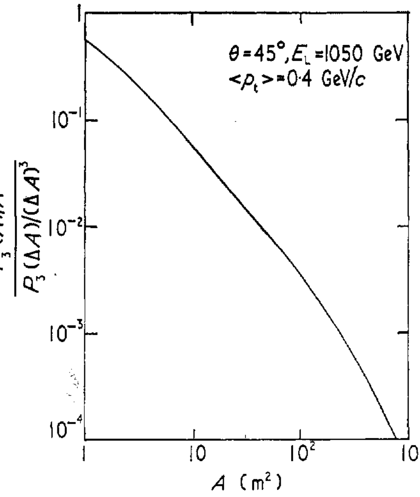

Figure 4. Area-dependence of the probability of detecting three muons (after II). $P_3(A)$ is the probability of detecting three muons in an area A.

The effect of finite detector area on the rates of twos and threes can be seen in figures 3 and 4, which come from the work reported in II. Although they refer to specific values of muon threshold energy E_{LO}, angle θ_{O} and $\langle p_{\mathrm{TO}} \rangle$, they may be used for other conditions $E_{\mathrm{L}\theta}$ and $\langle p_{\mathrm{T}} \rangle$ if we replace A by FA where

$$F = \frac{S_0(\theta_0, E_{\mathrm{LO}}, \langle p_{\mathrm{TO}} \rangle)}{S(\theta, E_{\mathrm{LO}}, \langle p_{\mathrm{T}} \rangle)} = \left(\frac{\langle p_{\mathrm{TO}} \rangle \sec^{1\cdot 3}\theta_0 \, E_{\mathrm{L}\theta}{}^{0\cdot 8}}{\langle p_{\mathrm{T}} \rangle \sec^{1\cdot 3}\theta \, E_{\mathrm{LO}}{}^{0\cdot 8}} \right)^2$$

The apertures for twos and threes corrected for the relative size of the detector and the shower and adjusted to an effective sensitive area A_0 can thus be derived in a straightforward manner.

2.4. *Multiple muon 'intensities'*

Arrays of effective apertures were calculated as a function of zenith and depth. $A_{0ik'}$ was taken as 20 m^2 (the actual areas varying from about 1 m^2 to 60 m^2) and

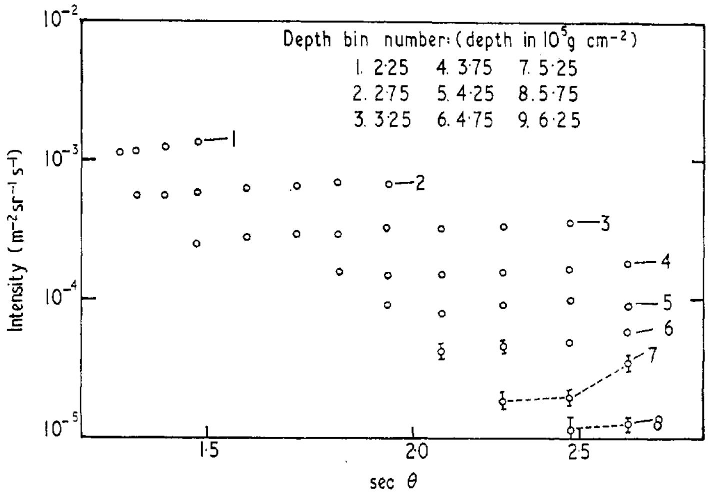

Figure 5. Intensity against sec θ for single muons.

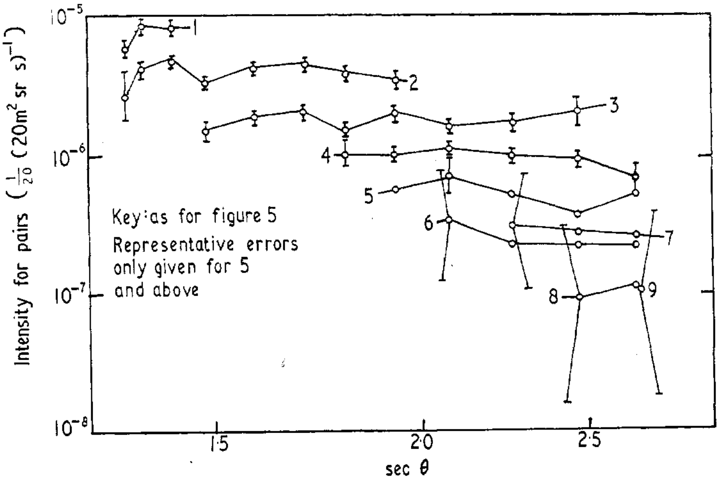

Figure 6. 'Intensity' against sec θ for twos relating to a standard area of 20 m^2.

$\langle p_T \rangle$ was alternately taken to be 0·40 GeV/c and 0·80 GeV/c. Slant depths were converted to energies using the range energy relation with $b = 4 \times 10^{-6}$ g^{-1} cm^2.

The 'intensities' obtained are plotted in figures 5, 6 and 7. All the uncertainties indicated are statistical and result from the count of the uncorrected numbers of events presented in tables 1, 2 and 3.

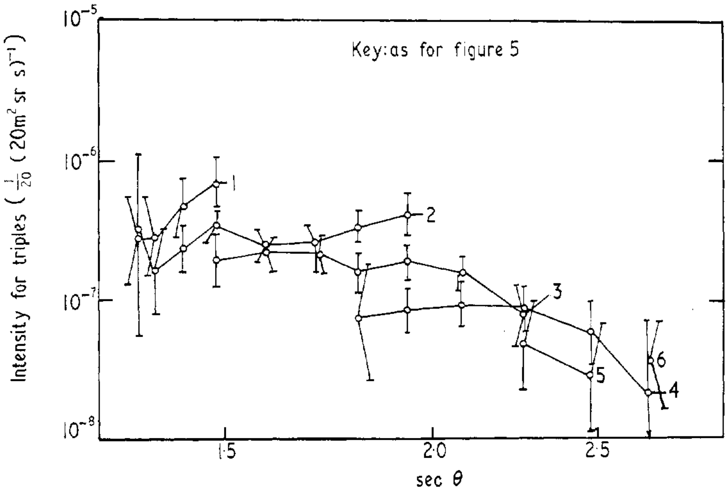

Figure 7. 'Intensity' against sec θ for threes relating to a standard area of 20 m^2.

3. Discussion

3.1. *Critique of the method of analysis*

A number of factors may affect the accuracy of the derived intensities. Chief among these are: detector triggering efficiency, the depth and zenith-angle dependence of the rates coupled with the geometry of the detector, the rock composition, and the accuracy of the event recognition program.

In the present experiment the triggering efficiency of the detector was measured continuously as the data were taken, and checks were made to compare recent runs with previous ones to look for any long-term systematic effects. None was found.

Intensities for events of all multiplicities were corrected for the distributions of contributing depths within a given depth bin to the intensities corresponding to the depth at the centre of the bins. This was done using the ratios of the single muon vertical depth intensity (see § 2) as a first approximation. The approximation is based on the two following empirical observations. First, measured muon intensities at the same zenith angles and slant depths but at different azimuth angles are very nearly the same. Second, the relative attenuation of the intensities with depth at fixed zenith follows roughly that of the vertical muon depth intensity. These positive results argue, then, for the essential correctness of the adjustments made in the individual 5×10^4 g cm^{-2} depth bins; in fact these adjustments were in no case large.

Intensities of twos and threes were corrected for the effect of the relative size of muon showers underground to the detector size in the manner outlined in § 2. The bases for this analysis are probabilities for seeing two and three muons in a square area which result from a numerical calculation using the radial density distribution of paper II. Although the Utah detector is large, it is still small compared with the extent of muon showers which are spread out over an area of 100 to 300 m^2 for the conditions of the present experiment. The detector area is not, in general, square, but it has been shown by direct computation that the probabilities calculated are relatively insensitive to the exact shape of the detector area.

The result that essentially the same intensities are obtained for twos and threes whether the apertures are corrected assuming that $\langle p_T \rangle = 0{\cdot}4$ GeV/*c* or that $\langle p_T \rangle = 0{\cdot}8$ GeV/*c* does not imply that the mean transverse momentum of the shower muons is not an important factor determining the multiple muon intensities. On the contrary, the absolute magnitudes of these intensities predicted for a given flux of primary cosmic rays is strongly affected by this choice. The important fact is that the corrections being made for both the area in question and the standard area of 20 m^2 are usually significant but their ratio does not depend sensitively on $\langle p_T \rangle$ for this range.

Because of the data combination scheme adopted, the results obtained depend on the density and value of Z^2/A assigned to the rock. Handbook values for our local formations based on widely distributed surface samples give a weighted average for the density of 2·47 g cm^{-2} as compared with a value of 2·61 g cm^{-2} obtained from measurements of samples taken from underground tunnel areas. The best value for the mean density of rock has been taken to be the mean of these two values: 2·54 g cm^{-2}. The weighted average of Z^2/A was found to be 5·65 and this value was used to correct the depth values to that of standard rock ($Z^2/A = 5{\cdot}5$), a correction of only 0·5 to 2% over our range of depths.

The differences between the best mean value of the rock density and the handbook value on the one hand and the underground samples on the other is 5% and it is unlikely that we would have to contend with greater deviations than this. We would expect qualitatively that a decreased density would decrease the dependence of the data on sec θ while increasing the rock density would increase the dependence. The data have been rebinned assuming deviations from the best value and these expectations are confirmed. A spread in densities of more than 10% is required to give a significant effect and this is considered to be most unlikely.

The event recognition program used to select the events for intensity analysis has been exhaustively checked by hand scanning several thousand events, the errors in event identification were found to be less than 1%.

Acknowledgments

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References

Adcock, C., Coats, R. B., Wolfendale, A. W., and Wdowczyk, J., 1971, *J. Phys.* A: *Gen. Phys.*, **4**, 276–90.

Bergeson, H. E., *et al.*, 1967, *Phys. Rev. Lett.*, **19**, 1487–91.

—— 1968, *Phys. Rev. Lett.*, **21**, 1089–93.

Porter, L. G., and Stenerson, R. A., 1969, *J. Phys.* A: *Gen. Phys.*, **2**, 374–91.